# Nonlinearities and instabilities of dissipative drift waves in dusty plasmas

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A linear and nonlinear description of drift waves in a dusty plasma is given. Equations describing the simultaneous change of electron density, drift potential, and the charges on dust particles are obtained. They constitute a generalization of Hasegawa-Wakatani equations to dusty plasmas. It is shown that the main effect of dust is the enhancement of the nonadiabaticity of the system, implying strong effects on the transport process.

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## I. INTRODUCTION

Recently there has been a great deal of interest in studying the linear and nonlinear evolution of instabilities in dusty plasmas [1-4]. The presence of dust in the plasma modifies its collective properties. Dust particles are highly charged and usually have a size much less than the plasma Debye length (the charge of the dust particle $Z_d$ in units of the electron charge can be of the order of $10^4$ or even higher). Dust particles are often negatively charged by plasma currents [5]. An important consequence of the dynamical charging of dust particles is the appearance of new collective modes and the modification of the plasma dielectric properties [3,4]. In particular, the modes with frequencies much less than the characteristic dust collisional charging frequency are substantially changed. Therefore, low frequency drift modes can be actively affected by dust. Recently, using a kinetic approach, we have estimated the growth rate of this new type of drift instability, driven by the process of dust charging [4]. We found that this growth rate is comparable to or much larger than the usual drift instability one, even for a low dust density. In this paper, we continue this investigation using a hydrodynamic approach and consider the opposite limit when the dissipation processes are not determined by the kinetic Landau damping but by dust-particle collisions. The dissipative drift waves can play an important role in the ionospheric plasma turbulence, in astrophysical and cometary plasma as well as in edge plasma turbulence of tokamaks [6,7,4]. The origin of dust in the scrape-off layer of the tokamak can be due to the plasma-wall interaction while in the ionosphere it has many sources: pollution, vulcanic eruptions, meteorites, . . . etc. In this paper, we will give a generalization of the dissipative drift wave equations taking into account the main nonlinearities that are already present in the Hasegawa-Wakatani equations (HW) without dust [8].

The presence of dust introduces a high rate of collisions of plasma particles with dust. Nevertheless, the distribution of the dust particles can be considered as fixed since the drift frequency is much higher than any frequency related to the dust motion. Thus the dust can be treated kinetically neglecting the dust-dust collisions.

There exist two possible descriptions of the system of charged particles: a hydrodynamic and a kinetic description. These two approaches are complementary. For a process in which the characteristic time is much larger than the time scale of binary particle collisions and for which the characteristic length is much larger than the mean free path due to binary collisions, the hydrodynamic approach is normally used. Under the opposite conditions, the kinetic approach is used. In the hydrodynamic description it is usually supposed that the collisions have sufficient time to establish the local thermal distribution of particles. However, the deviation from the latter is due to the presence of an average velocity, density and temperature inhomogeneities or to a slow time dependence. The presence of dust particles can change this separation between the hydrodynamic and the kinetic approach if dust particles are highly charged and if their density is sufficiently high. It often occurs that the collision frequency of plasma particles with dust can be much larger than the frequency of binary plasma particle collisions. Let us mention that if the collisions are dominated by plasma-dust interactions, the plasma-particle distribution is determined mainly by these collisions and not by binary plasma-particle collisions as in usual hydrodynamics. Then the binary plasma-particle collisions can be considered as a secondary effect or can even be neglected in the first approximation. Thus the concept of hydrodynamics for slow motions of charged plasma particles is changed since binary collisions are not the dominating interaction. Transport coefficients such as viscosity, thermal conductivity, diffusion, and other usual quantities will then for slow motions of plasma be either modified or can be neglected as is the case for viscosity that is small compared to the friction of the plasma particles on dust. The main qualitative effect is that the plasma becomes more dissipative as compared to the case when the dust is absent. This large dissipation implies that all dissipative instabilities can be enhanced in the presence of the dust particles.

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We will deal in this paper, only with a specific case of dissipative instabilities: the dissipative drift wave instability in dusty plasmas. We shall consider the case when the drift waves frequencies are less than the frequency of plasma particle–dust collisions; and wave numbers of drift waves are less than the inverse mean free path of plasma-particle-dust collisions.

The main dissipative process introduced by the presence of dust particles is their charging by plasma currents in which the plasma particles disappear from the plasma since they can be attached to the dust. This process can be considered as a constant sink of plasma particles or, in other words, their recombination on dust. The latter process takes place when an equilibrium in the dust charge is reached and when currents of electrons and ions on dust are equal. Usually dusty plasmas are stationary and the losses of plasma particles due to their recombination on dust particles are compensated by some ionization processes or by plasma-particle fluxes from the regions where dust is absent to the regions where it exists. The latter case is realized in planetary rings and probably in the ionosphere. Due to the presence of external sources of plasma particles, the plasma-dust system could be considered as an open system where the self-organization phenomena and the associated coherent structures are of general interest.

This work is organized as follows. Section II starts with some estimates of effective collision frequencies showing that the rate of the plasma-particle collisions with dust in most relevant applications is substantially larger than the rate of the binary plasma-particle collisions. We then calculate the effective collision frequency for plasma particles in the continuity equation and in the momentum equation. This calculation is performed for the arbitrary plasma-particle distribution at equilibrium. Section III deals with the basic assumptions used in the description of the dissipative drift waves in dusty plasmas. In Sec. IV, we derive the nonlinear equations for the dissipative drift waves in dusty plasmas and in Sec. V we study their linear stability.

## II. EFFECTIVE COLLISION FREQUENCIES OF PLASMA PARTICLES WITH DUST

The rate of the dissipative charging process can be estimated by the charging frequency $\nu_{\rm ch}$ that is the inverse of the time needed to charge a single dust particle in a plasma by plasma currents. This frequency can be estimated from the usual limiting orbit approximation for the cross sections of the charging processes [2]. The charging frequency appears to be approximately $Z_d$ times larger than the binary ion-ion collision frequency. The values of $Z_d$ are in most cases of very large interest of the order of $10^3$ or $10^4$ or even higher. Thus $\nu_{\rm ch}$ is indeed much larger than the binary ion-ion collision frequency [2,4]. The rate of ion charging collisions is proportional to the dust density $n_d$ and differs from $\nu_{\rm ch}$ by a dimensionless parameter $P/(1+P)$ where $P=(n_d Z_d)/n_e$ and $n_e$ is the electron plasma density. In the experiments, the value of $P$ is of the order of unity. The frequency of the ion-dust charging collisions $\nu_{\rm id}^{\rm ch}$, in which the ions are lost recombining with electrons on dust particles is thus of the order of $P\nu_{\rm ch}$for $P \ll 1$ and is of the order of $\nu_{\rm ch}$ for $P \gg 1$. The rate of the elastic collisions of the ions with the dust particles (when the ions are deflected by the dust) differs from the rate of the charging collisions of the ions on dust (when the ions are attached to the dust particle) by a factor of the order of $\ln(d/a)$ for $d \gg a$, where $d$ is the Debye screening length in a plasma and $a$ is the radius of the dust particle. This estimate shows that indeed both the charging and the elastic collision frequencies of ions on the dust particles are much larger than the frequency of the binary ion-ion collisions. The rate of the charging collisions of the electrons on the dust particles is $1+P$ times larger than the rate of ion-dust charging collisions. The rate of the elastic electron dust is even larger by a factor $[m_i/m_e(T_i/T_e)^3]^{1/2}$. Thus for both electrons and ions their collision rates with dust particles will dominate the rate of their binary collisions. In what follows we will neglect the plasma-particle binary collisions and will take into account only the collisions of plasma particles with dust.

One should distinguish the effective collision frequencies of the plasma particles with the dust that enters in the momentum equations of the plasma particles and the effective collision frequencies of the plasma particles with dust entering in the continuity equations of the plasma particles. We denote the first by $\tilde{\nu}_{e,d}$ and $\tilde{\nu}_{i,d}$ and the second by $\overline{\nu}_{e,d}$ and $\overline{\nu}_{i,d}$ and for electrons and ions, respectively. In the calculations of these collision frequencies we take into account both the charging and the elastic collisions. We suppose that the particle distribution is slightly anisotropic. This anisotropy is due to the existence of an average particle drift $\mathbf{u}$. We suppose also that there exists a small variation of plasma particle densities $\delta n_{ei} \ll n_{0ei}$ where $n_{0ei}$ is the equilibrium density of the plasma particles. The distribution functions of electrons and ions can then be written as follows:

$$f_{e,i}(\mathbf{v})=f_{e,i}^{eq}(|\mathbf{v}-\mathbf{u}|)+\frac{\delta n_{e,i}}{n_{0,e,i}}f_{e,i}^{eq}(v) = \left[1+\frac{\delta n_{e,i}}{n_{0,e,i}}\right]f_{e,i}^{eq}(v)-\frac{\mathbf{u}\cdot\mathbf{v}}{v}\frac{\partial}{\partial v}f_{e,i}^{eq}(v) , \tag{1}$$

where $v=|\mathbf{v}|$, and $f_{e,i}^{eq}(v)$ is the equilibrium distribution function that is determined by the balance of the plasma-particles creation by an external source and their recombination on dust particles. We suppose the particle distribution function $f_{e,i}^{eq}(v)$ to be isotropic. In dusty plasmas, this distribution can depend on the source of the plasma particles and on the charging process on the dust particles, i.e., it cannot be a Maxwellian thermal distribution. For example, in the case where the charge of the dust particles is negative, only fast electrons will be able to take part in the charging process and the tail of the electron distribution function is changed in such a way as to decrease the relative number of fast electrons. Thus we will consider $f_{e,i}^{eq}(v)$ to be arbitrary. To give some estimate of the collision frequencies, we shall also consider the case where $f_{e,i}^{eq}(v)$ is Maxwellian. For other distribution functions, one should keep in mind, that in the expressions we obtain for the thermal distributions, the average particle velocity should be substituted for the thermal particle velocity. The particle momentum in a unit plasma volume can be written as

$$\mathbf{P}_{e,i}=\int m_{e,i}\mathbf{v}f_{e,i}(\mathbf{v})d^3v \ . \tag{2}$$

We denote the change of the plasma-particle momenta due to collisions with the dust particles as $\delta\mathbf{P}_{e,i}$ and define the effective collision frequencies $\tilde{\nu}_{i,e,d}$ by the relation

$$\frac{d}{dt}\delta\mathbf{P}_{e,i}=-\tilde{\nu}_{e,i,d}n_{0,e,i}m_{e,i}\mathbf{u} \ . \tag{3}$$

This expression should be added in the equation of the change of the particle average momenta, containing the usual $(\mathbf{u}\cdot\nabla)\mathbf{u}$ term, the Lorentz force term and the gradient pressure term. The $\tilde{\nu}_{e,i,d}$ wills consist of two parts: The first term is related to the charging plasma particle–dust collisions $\tilde{\nu}^{\text{ch}}_{e,i,d}$ and the second term is related to the elastic plasma particle–dust collisions $\tilde{\nu}^{el}_{e,i,d}$

$$\tilde{\nu}_{e,i,d}=\tilde{\nu}^{\text{ch}}_{e,i,d}+\tilde{\nu}^{el}_{e,i,d} \ . \tag{4}$$

We start with the calculation of the effective frequency of elastic collisions. We will use for that the Landau collision integral with the maximum value of the impact parameter to be the Debye screening length $d$ and the minimum value of the impact parameter to be the radius of the dust particle (supposing that all dust particles are of the same size and supposing that the size of the dust particles is much less than the Debye screening radius as is the case in most applications). Then

$$\frac{d}{dt}\delta\mathbf{P}_{e,i}=-\frac{2\pi Z_d^2 n_d e^4}{m_{e,i}}\ln\frac{d}{a}\int\mathbf{v}\frac{\partial}{\partial v_i}\frac{1}{v}\left[\delta_{i,j}-\frac{v_iv_j}{v^2}\right]\times\frac{\partial}{\partial v_j}\frac{\mathbf{u}\cdot\mathbf{v}}{v}\frac{\partial}{\partial v}f^{eq}_{e,i}(v)d^3v \ . \tag{5}$$

The integration over the angles of the last expression gives

$$\frac{d}{dt}\delta\mathbf{P}_{e,i}=\frac{4\pi Z_d^2 n_d e^4}{3m_{e,i}}\mathbf{u}\ln\frac{d}{a}\int\frac{1}{v^2}\frac{\partial}{\partial v}f^{eq}_{e,i}(v)d^3v \ . \tag{6}$$

A comparison of this expression with the definition of the collision frequency (3) leads to

$$\tilde{\nu}^{el}_{e,i,d,}=\frac{4\sqrt{2\pi}Z_d^2n_de^4}{3m^2_{e,i}\bar{v}^3_{e,i}}\ln\frac{d}{a} \ , \tag{7}$$

where the average velocity $\bar{v}_{e,i}$ is defined by the relation

$$\frac{1}{\bar{v}^3_{e,i}}=-\left[\frac{\pi}{2}\right]^{1/2}\frac{\int\frac{1}{v^2}\frac{\partial}{\partial v}f^{eq}_{e,i}(v)d^3v}{\int f^{eq}_{e,i}(v)d^3v} \ . \tag{8}$$

For a Maxwellian thermal distribution the $\bar{v}_{e,i}$ coincides with the average thermal velocity $v_{T,e,i}=\sqrt{T_{e,i}/m_{e,i}}$.

To compute the effective charging collisions frequencies one needs to know the cross sections of the charging of dust by plasma currents. For the ballistic approximation [2], these cross sections are given by

$$\sigma^{\text{ch}}_{i,e,}(v)=\pi a^2\left[1\pm\frac{2Z_de^2}{m_{i,e,}v^2a}\right] . \tag{9}$$

The change in the plasma-particle momenta due to the charging collisions will be

$$\frac{d}{dt}\delta\mathbf{P}_{e,i}=-m_{e,i}\int\mathbf{v}n_dv\sigma^{\text{ch}}_{i,e}(v)f^{eq}_{e,i}(|\mathbf{v}-\mathbf{u}|)d^3v \ . \tag{10}$$

We then change the integration $\mathbf{v}\to\mathbf{v}+\mathbf{u}$, expand the result on $\mathbf{u}$ and average on angles. This gives

$$\tilde{\nu}^{\text{ch}}_{e,i}=\frac{\int\left[1+\frac{1}{3}v\frac{\partial}{\partial v}\right]n_dv\sigma^{\text{ch}}_{i,e,}(v)f^{eq}_{e,i}(v)d^3v}{\int f^{eq}_{e,i}(v)d^3v} \ . \tag{11}$$

In the case where the equilibrium dust charge is determined by the balance of electron and ion currents on dust particles and the equilibrium particle distribution is Maxwellian, one can express both the elastic and the charging frequencies through the following three parameters,

$$z=\frac{Z_de^2}{aT_e},\quad \tau=\frac{T_i}{T_e},\quad P=\frac{n_dZ_d}{n_e} \ , \tag{12}$$

which are not independent since they should satisfy the current balance equation

$$\exp(-z)=\left[\frac{m_e}{m_i}\right]^{1/2}\frac{1+P}{\sqrt{\tau}}(\tau+z) \ . \tag{13}$$

For applications it is useful to express the effective collision frequencies in terms of the charging frequency $\nu_{\text{ch}}$. Using Eq. (13) we get

$$\nu_{\text{ch}}=\frac{\omega^2_{pi}a}{v_{Ti}\sqrt{2\pi}}(1+\tau+z) \ , \tag{14}$$

where $\omega_{pi}=[(4\pi n_{0i}e^2)/m_i]^{1/2}$ is the ion plasma frequency and $v_{Ti}=\sqrt{T_i/m_i}$ is the ion thermal velocity. For thermal distributions of plasma particles the expressions (7) and (11) give

$$\tilde{\nu}^{el}_{e,d}=\nu_{\text{ch}}\left[\frac{2P\exp(z)}{3z(1+\tau+z)}(\tau+z)\ln\frac{d}{a}\right] , \tag{15}$$

$$\tilde{\nu}^{el}_{i,d}=\nu_{\text{ch}}\left[\frac{2P}{3z(1+\tau+z)\tau(1+P)}\ln\frac{d}{a}\right] , \tag{16}$$

$$\tilde{\nu}^{\text{ch}}_{e}=\tilde{\nu}^{\text{ch}}_{i}\left[\frac{(1+P)(\tau+z)(4+z)}{z+\frac{4\tau}{3}}\right]=\nu_{\text{ch}}\frac{P(\tau+z)(4+z)}{z(1+\tau+z)} \ . \tag{17}$$

According to Eq. (13) the exponential term in (15) is usually large and the rate of the elastic electron-dust collisions is usually much larger than the rate of the ion-dust elastic collisions. The rates of the charging collisions for electrons and ions could be of the same order of magnitude.

Now let us consider the effective collision frequencies of the plasma particles with dust that enters in the continuity equation for electron and ion densities. It is easy to find that the elastic collisions conserve the number of particles and therefore they do not contribute in the continuity equation.

For the change of distribution function of the plasma particles due to the charging collisions we use the expression

$$\frac{df_{e,i}(\mathbf{v})}{dt}=-n_d\sigma_{i,e}^{\rm ch}(v)v[f_{e,i}(\mathbf{v})-f_{e,i}^{eq}(v)] \ . \tag{18}$$

The last term in the right-hand side (rhs) of this equation describes the source of the plasma particles which compensates their losses due to the recombination on the dust particles. We define the effective collision frequency $\bar{\nu}_{e,i,d}$ by the equation

$$\frac{dn_{e,i}}{dt}=\frac{\partial n_{e,i}}{\partial t}+\nabla\cdot(n_{e,i}\mathbf{u})=-\bar{\nu}_{e,i,d}(n_{e,i}-n_{0,e,i}) \ . \tag{19}$$

From Eq. (18) we find then

$$\bar{\nu}_{e,i,d}=\frac{\int n_d v\sigma_{i,e}^{\rm ch}(v)f_{e,i}^{eq}(v)d^3v}{\int f_{e,i}^{eq}(v)d^3v} \ , \tag{20}$$

which in the case of thermal Maxwellian plasma-particle distributions becomes

$$\bar{\nu}_{e,d}=\bar{\nu}_{i,d}(1+P)=\nu_{\rm ch}\frac{P(\tau+z)}{z(1+\tau+z)} \ . \tag{21}$$

These frequencies can thus be of the same order of magnitude for electrons and ions. It is easy to see that the first relation of (21) is a consequence of the equality of the equilibrium fluxes of electrons and ions on dust particles.

## III. BASIC ASSUMPTIONS IN THE DESCRIPTION OF DISSIPATIVE DRIFT WAVES IN DUSTY PLASMAS

We derive the linear and nonlinear equations for the dissipative drift waves in a dusty plasma under certain assumptions. First, we will suppose that the frequency of the drift waves and all the effective collision frequencies of the plasma particles with the dust particles derived in the previous section are much less than the ion-cyclotron frequency otherwise the drift waves will be completely suppressed by the collisions.

$$\omega_{ci}=\frac{|e|B_0}{cm_i}\gg\bar{\nu}_{e,i,d},\bar{\nu}_{e,i,d}, \ . \tag{22}$$

We also suppose that the effective collision frequency of the plasma particles on dust is much larger than the frequency of the binary collisions of the plasma particles. Therefore we can neglect the effects of the binary collisions such as the viscosity or diffusion. Indeed these quantities are small as compared to the friction of the plasma particles on dust.

Second, we suppose that the vector nonlinearities dominate the scalar nonlinearities. Physically, this assumption means that the gradients of the functions (for example, the wave potential) can be of the order of the ion Larmor radius for electron temperature, ($\rho_s=c_s/\omega_{ci}$). These assumptions are usually made in the derivation of the Hasegawa-Mima and HW nonlinear equations for drift waves in the absence of dust [8]. The ordering in the case of the latter equations shows that the dimensionless potential ($\Psi=e\varphi/T_e$) is of the order of the ratio of the drift frequency to the ion-cyclotron frequency. This means that the term $\Psi^2$ times the drift frequency which appears in the expansion of the exponential term describing the adiabatic response of electrons in these equations will be relatively small as compared to the term containing the fourth power of the gradient of the potential and describing the vector nonlinearity. In the case of a dusty plasma, the behavior of the nonlinear dissipative drift waves cannot be described by a single dimensionless potential as is the case for the Hasegawa-Mima equation (without dust) or by two dimensionless parameters (density and potential) as in the case of the (HW) equations. We need to introduce three parameters. The third parameter describes the change of dust charges. Since the drift waves modify the distributions of both the electrons and ions taking part in the currents on dust particles, the charges of dust particle will be changed. However, due to the possible delay in the charging process, a phase shift arises between the plasma potential and the charge variation. We describe this effect by the charging equation that will couple the charge changes on dust particles with the density variations in drift waves. On the other hand, the density variation in drift waves will also depend on the dust charges through the effective collision frequencies of the plasma particles with dust. Thus we need to find the coupled equations for all three parameters describing the dissipative drift waves in a dusty plasma. Let us note that the scalar products of all combinations of the three parameters will enter the scalar nonlinearities. We will, as in the absence of dust, consider only the quadratic nonlinearities, thus neglecting the cubic ones. We are then considering only the quadratic combinations of these three parameters. The ordering similar to the one used in deriving the Hasegawa-Mima equation shows, in the presence of dust, that all these scalar nonlinearities are of the order of the ratio of the effective collision frequency of the plasma particles to the ion-cyclotron frequency or again of the order of the ratio of the drift frequency to the ion-cyclotron frequency. We should also restrict ourselves only to the case of electrostatic drift waves. Thus the second assumption can be formulated in a more precise way as following. The drift waves are electrostatic with only the quadratic nonlinearities and the scalar quadratic nonlinearities are neglected as compared to the vector nonlinearities.

Third, we will suppose that the quasineutrality condition for the drift waves is valid but modified by the presence of the change in the dust charges. We will also suppose that the frequency of the drift waves is sufficiently large as compared to the plasma-dust frequency. The dust is considered immobile and only the charges on the dust particles are changed in the drift waves. The latter assumption is valid in most applications since the masses of dust particles are very large.

After having described the assumptions used in the derivation of the dissipative drift wave equations, let us now introduce the variables which will be used in the description of the drift waves in this paper. First of all we should mention that the quasineutrality condition in the absence of drift waves is changed by the presence of dust due to the rather big charges on the dust particles. We should write for the case of negatively charged dust particles

$$n_{0i}=n_{0e}+n_d Z_d \ . \tag{23}$$

This relation can be also written using the parameter $P$ (Eq. 12) as follows:

$$n_{0i}=n_{0e}(1+P) \ , \tag{24}$$

where $n_{0e}$ and $n_{0i}$ are the equilibrium values for the electron and ion densities, respectively. A consequence of Eq. (24) is that the electron and ion drift frequencies are not equal. These frequencies $\omega^*_{e,i}$ are defined by

$$\omega^*_{e,i}=-\frac{c_s^2}{\omega_{ci}}k_y\frac{1}{n_{0e,i}}\frac{\partial}{\partial x}n_{0e,i} \ , \tag{25}$$

where $k_y$ is the wave vector component perpendicular to both the magnetic field and the density gradient.

The relation between these frequencies is derived from Eq. (23) and contains the gradient of the dust charge density

$$\omega^*_i=\omega^*_e\frac{1}{1+P-P\dfrac{\partial \ln n_d Z_d}{\partial \ln n_{0i}}} \ . \tag{26}$$

We introduce then the dimensionless variables which will describe the drift waves. These variables will be the charge variation on the dust particles, the ion and the electron density variations, and the electrostatic potential

$$\frac{\delta Z_d}{Z_d}, \quad \frac{\delta n_i}{n_{0i}}, \quad \frac{\delta n_e}{n_{0e}}, \quad \Psi=\frac{e\varphi}{T_e} \tag{27}$$

where $\varphi$ is the electrostatic potential of the drift waves and $\delta Z_d$ is the variation of charges on the dust particles. Note that the frequencies of the plasma-particle dust collisions depend on the total charge on the dust particles including $\delta Z_d$. Expressions (7), (11), and (20) contain indeed the total charge. But expressions (15), (16), (17), and (21) were obtained from expressions (7), (11), and (20) by substituting in the value of the equilibrium charge. The additional terms due to $\delta Z_d$ describe additional nonlinearities which will be the main subject of calculation in the next section.

The quasineutrality condition for the changes in the electron density, the ion density, and the dust charges in drift waves can be considered as a relation between the variables introduced in (27) $n_i=n_{0i}+\delta n_i$, $n_e=n_{0e}+\delta n_e$:

$$\delta n_i=\delta n_e+\delta Z_d n_d \ . \tag{28}$$

Dividing this relation by expression (23) we find

$$\frac{\delta n_i}{n_{0i}}=\frac{\delta n_e}{n_{0e}}\frac{1}{1+P}+\frac{\delta Z_d}{Z_d}\frac{P}{1+P} \ . \tag{29}$$

## IV. NONLINEAR EQUATIONS FOR DISSIPATIVE DRIFT WAVES

We start with the electron momentum equation in which we neglect the electron inertia

$$0=-v_{Te}^2\nabla n_e+v_{Te}^2 n_e\nabla\Psi-\omega_{ce}n_e\mathbf{u}_e\times\mathbf{z}-n_e\tilde{\nu}_{ed}\mathbf{u}_e \ , \tag{30}$$

where $\mathbf{z}=\mathbf{B}_0/B_0$, $\omega_{ce}=(eB_0)/m_e c)$. The equilibrium solution of this equation is the usual diamagnetic electron drift with the velocity $\mathbf{u}_{e0}$

$$\mathbf{u}_{e0}=-\frac{v_{Te}^2}{\omega_{ce}n_{0e}}\mathbf{z}\times\nabla n_{0e} \ . \tag{31}$$

We expand Eq. (30) in a small parameter $\varepsilon$ to solve it for the perturbations, $\varepsilon=c_s/(L_n\omega_{ci})$ where

$$\frac{1}{L_n}=(1/n_{0e})(\partial/\partial x)n_{0e} \ .$$

The first and second order terms of this expansion (in the case where the scalar nonlinearities are neglected) can be written in the form

$$\delta\mathbf{u}_{e\perp}=\frac{v_{Te}^2}{\omega_{ce}}\left[\mathbf{z}\times\nabla\Psi-\mathbf{z}\times\delta\frac{1}{n_e}\nabla n_e\right] , \tag{32}$$

$$\delta u_{e,z}=\frac{v_{Te}^2}{\tilde{\nu}_{ed}}\frac{\partial}{\partial z}\left[\Psi-\frac{\delta n_e}{n_{0e}}\right] . \tag{33}$$

We should emphasize that the electron inertia can be neglected for disturbances perpendicular to the magnetic field. For the disturbances parallel to the magnetic field, the electron inertia can be taken into account if the effective collision frequency of the electrons with the dust is comparable to the frequency of the waves. Then one should change the frequency $\tilde{\nu}_{ed}$ to $(\tilde{\nu}_{ed}+\partial/\partial t)$ in Eq. (33).

In the momentum equations for ions we take into account the ion inertia and we neglect the ion pressure for the perpendicular motion. The equation for the ion momentum is

$$m_i n_i\left[\frac{\partial}{\partial t}\mathbf{u}_i+\mathbf{u}_i\cdot\nabla\mathbf{u}_i\right]=-en_i\nabla\Psi-T_i\nabla n_i+\frac{en_iB_0}{c}\mathbf{u}_i\times\mathbf{z}-m_i n_i\tilde{\nu}_{id}\mathbf{u}_i \ . \tag{34}$$

We solve Eq. (34) using the expansion on the same parameter $\varepsilon$. In the first approximation we get the following term representing the electric drift:

$$\mathbf{u}_{\perp i}=\frac{c_s^2}{\omega_{ci}}\mathbf{z}\times\nabla\Psi \ . \tag{35}$$

The second order terms of this expansion are the ion polarization drift

$$\delta\mathbf{u}_{\perp i}=-\frac{c_s^2}{\omega_{ci}}\frac{\partial}{\partial t}\nabla_\perp+\frac{c_s^4}{\omega_{ci}^3}[\mathbf{z}\times(\mathbf{z}\times\nabla\Psi\cdot\nabla)\mathbf{z}\times\nabla\Psi] \tag{36}$$

and the term associated with the parallel current,

$$\delta u_{zi}=\frac{c_s^2}{\bar{\nu}_{id}}\left[\frac{\partial}{\partial z}\Psi+\tau\frac{\delta n_i}{n_{0i}}\right] . \tag{37}$$

If we take into account the ion inertia, we should then change the frequency $\bar{\nu}_{id}$ to $[\bar{\nu}_{id}+(\partial/\partial t)]$ in Eq. (37) similarly to what was done for the electrons in Eq. (33).

Now we can deduce the nonlinear equations. The electron continuity equation gives by using the expansion on the parameter ε the following relation:

$$\frac{\partial}{\partial t}\delta n_e+\delta\mathbf{u}_{e\perp}\cdot\nabla_\perp n_{0e}+\nabla_\perp\cdot\delta n_e\delta\mathbf{u}_{e\perp}+\mathbf{u}_{0e}\cdot\nabla_\perp\delta n_e +n_{0e}\frac{\partial}{\partial z}\delta u_{ez}=-\delta\bar{\nu}_{ed}n_{e0}-\bar{\nu}_{ed}\delta n_e , \tag{38}$$

where it was taken into account that $\nabla_\perp\cdot\mathbf{u}_{e0}=0$. The $\delta\bar{\nu}_{ed}$ in Eq. (38) describes the change in the charging frequency when the charge on the dust particle deviates from its equilibrium value. From Eq. (21) we find $\{(\delta\bar{\nu}_{ed}/\bar{\nu}_{ed})=-z[\delta Z_d/Z_d]\}$. Using then the expressions (31), (32), and (33), we can write Eq. (38) in the form

$$\frac{\partial}{\partial t}\frac{\delta n_e}{n_{e0}}+\bar{\nu}_{ed}\left[\frac{\delta n_e}{n_{0e}}-z\frac{\delta Z_d}{Z_d}\right]=-\frac{v_{Te}^2}{n_{0e}\omega_{ce}}\mathbf{z}\times\nabla\Psi\cdot\nabla_\perp n_{0e} +\frac{v_{Te}^2k_z^2}{\bar{\nu}_{ed}}\left[\Psi-\frac{\delta n_e}{n_{0e}}\right] -\frac{v_{Te}^2}{\omega_{ce}}\nabla_\perp\frac{\delta n_e}{n_{0e}}\cdot\mathbf{z}\times\nabla\Psi . \tag{39}$$

Finally, we define the new dimensionless variables which take into account the small parameter ε

$$t\to\omega_{ci}\frac{\rho_s}{L_n}t,\quad x\to\frac{x}{\rho_s},\quad y\to\frac{y}{\rho_s},\quad \chi\to\frac{\delta Z_d}{Z_d}\frac{L_n}{\rho_s},\quad \Psi\to\Psi\frac{L_n}{\rho_s},\quad n\to\frac{\delta n_e}{n_{0e}}\frac{L_n}{\rho_s} \tag{40}$$

where $\rho_s=c_s/\omega_{ci}$ is the hybrid ion Larmor radius.

Thus with this normalization, we can find the relation

$$\frac{\partial n}{\partial t}+\alpha(n-z\chi)+\frac{\partial\psi}{\partial y}-\{n,\Psi\}=c_e(\Psi-n) , \tag{41}$$

where the coefficients $\alpha$ and $c_e$ are

$$\alpha=\frac{\bar{\nu}_{ed}}{\omega_{ci}\frac{\rho_s}{L_n}},\quad c_e=\frac{v_{Te}^2k_z^2}{\bar{\nu}_{ed}\omega_{ci}\frac{\rho_s}{L_n}} \tag{42}$$

and $\{.,.\}$ denotes the Poisson bracket

$$\{n,\Psi\}=\frac{\partial n}{\partial x}\frac{\partial\psi}{\partial y}-\frac{\partial n}{\partial y}\frac{\partial\psi}{\partial x} . \tag{43}$$

The following charging equation:

$$\frac{\partial}{\partial t}\frac{\delta Z_d}{Z_d}=-\nu_{\rm ch}\frac{\delta Z_d}{Z_d}+\frac{1+P}{P}\bar{\nu}_{id}\left[\frac{\delta n_e}{n_{0e}}-\frac{\delta n_i}{n_{0i}}\right] \tag{44}$$

can be written with dimensionless variables using expressions (29) and (40):

$$\frac{\partial}{\partial t}\chi+\beta\chi=\alpha(1+P)n , \tag{45}$$

where

$$\beta=\frac{\bar{\nu}_{id}}{\omega_{ci}\frac{\rho_s}{L_n}}\left[1+\frac{\nu_{\rm ch}}{\bar{\nu}_{id}}\right]=\alpha(1+P)\left[1+\frac{\nu_{\rm ch}}{\bar{\nu}_{id}}\right] . \tag{46}$$

Finally, with the same perturbation approach, the continuity equation for the ion can be written as

$$\frac{\partial}{\partial t}\delta n_i+\mathbf{u}_{i\perp}\cdot\nabla_\perp n_{0i}+\nabla_\perp\cdot\delta\mathbf{u}_{i\perp}n_{0i}+\nabla_\perp\cdot\delta n_i\mathbf{u}_{i\perp}+n_{0i}\frac{\partial}{\partial z}u_{iz} =-\delta\bar{\nu}_{id}n_{0i}-\bar{\nu}_{id}\delta n_i \tag{47}$$

with $\nabla_\perp\cdot\mathbf{u}_{i\perp}=0$.

The relations for the effective collision frequencies (21), for the density perturbation (29), for the ion velocities (35), (36), (37), and for the charging Eq. (44) allow us to write the ion continuity equation as follows:

$$\frac{\partial}{\partial t}\frac{\delta n_e}{n_{e0}}+\bar{\nu}_{ed}\left[\frac{\delta n_e}{n_{0e}}-z\frac{\delta Z_d}{Z_d}\right]+\frac{c_s^2}{n_{0i}\omega_{ci}}\mathbf{z}\times\nabla\Psi\cdot\nabla_\perp n_{0i}+\frac{c_s^2}{\omega_{ci}}\nabla_\perp\frac{\delta n_e}{n_{0e}}\cdot\mathbf{z}\times\nabla\Psi+\frac{c_s^2}{\omega_{ci}}P\nabla_\perp\frac{\delta Z_d}{Z_d}\cdot\mathbf{z}\times\nabla\Psi -\frac{c_s^2}{\omega_{ci}}(1+P)\nabla_\perp\cdot\frac{\partial}{\partial t}\nabla_\perp\Psi+\frac{c_s^4}{\omega_{ci}^3}(1+P)\nabla_\perp\cdot\{\mathbf{z}\times[(\mathbf{z}\times\nabla\Psi)\cdot\nabla]\mathbf{z}\times\nabla\Psi\} =\frac{c_s^2k_z^2}{\bar{\nu}_{id}}(1+P)\left[\Psi+\frac{\tau}{(1+P)}\frac{\delta n_e}{n_{0e}}+\frac{\tau P}{(1+P)}\frac{\delta n_e}{n_{0e}}\right] . \tag{48}$$

Then by using the normalization (40) and the relation (26) between the electron and ion drift frequencies, and by subtracting Eq. (41) from Eq. (48), we get the equation for the potential which together with the equation for the electron density (41) and the equation for the dust variation (45) constitute the new closed system of nonlinear equations describing the evolution of dissipative drift waves in dusty plasmas.

$$\frac{\partial}{\partial t}\nabla^2\Psi-\frac{PL_d^{-1}}{(1+P)(1+P-PL_d^{-1})}\frac{\partial\Psi}{\partial y}+c_i\left[\Psi+\frac{\tau}{1+P}n+\frac{\tau P}{1+P}\chi\right]-\frac{c_e}{1+P}(\Psi-n)=\{\nabla^2\Psi,\Psi\}-\frac{P}{1+P}\{\chi,\Psi\}\ , \tag{49a}$$

$$\frac{\partial n}{\partial t}+\alpha(n-z\chi)+\frac{\partial\Psi}{\partial y},-c_e(\Psi-n)=\{n,\Psi\}\ , \tag{49b}$$

$$\frac{\partial}{\partial t}\chi+\beta\chi=\alpha(1+P)n\ , \tag{49c}$$

where

$$c_i=\frac{c_s^2k_z^2}{\tilde{v}_{id}\omega_{ci}\dfrac{\rho_s}{L_n}}\quad\text{and}\quad L_d^{-1}=\frac{\partial lnn_dZ_d}{\partial lnn_{0i}} \tag{50}$$

are, respectively, the ion longitudinal motion parameter (adiabaticity parameter) and the dust density gradient.

This system differs from the usual HW system by the addition of a third equation for the dust charge variation and the presence of new linear and nonlinear terms depending on the dust parameters in the equations of potential and density (49a), (49b). The electron longitudinal motion coefficient $c_e$ in Eq. (49a) and (49b) does not take into account the binary plasma particle collisions as we have neglected them as compared to the dust particle collisions. As is usual in the (HW) system, the ion longitudinal motion parameter $c_i$ is smaller than the electron longitudinal parameter $c_e$ and in the further numerical investigations of the dissipative drift wave instability it will be neglected. We also neglect the viscosity term and the diffusion one which enter in the HW system since we neglect the binary collisions. It can be shown easily that within the hydrodynamical approach used in this paper the viscosity term produced by the dust-particle collisions is smaller than the friction of plasma particles on dust which is indeed taken into account. Let us remark that if the dust parameter $P$ vanishes and if we take into account the effects of binary collisions (diffusion and viscosity), we finally obtain the usual HW system. Note that if we keep the electron adiabaticity parameter $c_e$ fixed and vary the dust density by varying $P$, we find that in the limit of small $P$ we recover the HW system in the absence of diffusion and viscosity terms.

Another important remark concerning the system (49) is that the presence of dust decreases very substantially the value of the adiabaticity parameter $c_e$. Thus the system could behave almost adiabatically in the absence of dust and becomes nonadiabatic in the presence of dust.

## V. LINEAR STABILITY STUDY OF THE DISSIPATIVE DRIFT WAVE IN DUSTY PLASMA

The linear stability of the drift waves was already considered in a previous paper [4] for the case where the drift frequency is much larger than the effective collision frequency of the plasma particles with dust. In this case the kinetic effects (Landau damping) cause the nonadiabaticity in the longitudinal electron motion. In this paper we are considering the opposite case where the effective collision frequency is much larger than the drift frequency. Nonadiabaticity in the latter case is mainly determined by the electron-dust collisions.

To illustrate the change of nonadiabaticity due to the electron-dust collisions, we take, for instance, $\nu_{\text{ch}}P\approx 0.1\omega_{ci}$ and $P$ value of the order of 0.1, the dust gradient $L_d^{-1}$ of the order of one, and the parameters $\tau$ and $z$ close to 1. The longitudinal ion motion determined by the coefficient $c_i$ does not have a large influence on drift instability since approximately $(c_e\approx\sqrt{m_i/m_e c_i})$. The parameter $c_e$ describes the dissipation in the longitudinal electron motion and can be either $c_e\gg 1$ or $c_e\ll 1$, depending on the value of $k_z$. For $c_e\gg 1$ the electrons can be considered as almost adiabatically responding to the disturbances and $\Psi\approx n$ while for $c_e\ll 1$, the electrons behave nonadiabatically and the dissipation in longitudinal motion of electrons can drive the instability.

We need to emphasize two important consequences of the presence of dust in the plasma. The first is that the adiabaticity parameter $c_e$ is very much reduced. An approximate estimation of the reduction of the coefficient $c_e$ shows that for a dusty plasma there appears an additional factor $[1/(1+PZ_d)]$ in $c_e$. For the parameter values we gave above, this factor is $10^{-3}$. This means qualitatively that electrons which behave adiabatically in the absence of dust can behave nonadiabatically in the presence of dust. This effect is rather big and can lead to an appearance of an instability introduced by an increase of dissipation in the longitudinal electron motion, in the presence of dust. The physical reason for that, it that the dust charging process introduces a high rate of dissipation in the plasma. The second consequence of the presence of dust on the perpendicular motion is that dissipation is no longer produced by viscosity and diffusion, which together are related to binary collisions, but the major dissipation effect is the friction of the plasma particles on dust. The competition of these processes determines the rate of drift instability in dusty plasmas.

Figure 1 shows the comparison between the growth rates in the presence of dust and without it when the adiabaticity parameter is changing from the value 30 in absence of dust to the value 0.1 in the presence of dust and $P=0.1$. For the nondusty case, the computation takes into account the binary collision effects with a diffusion coefficient $D$ and viscosity $\nu$ such that $D=\nu=0.01$. In the presence of dust we take into account only the effects described by the system of Eq. (49). Let us note, that for this almost completely adiabatic case (in the absence of dust), the corresponding dusty plasma case presents a high rate of instability. The latter growth rate is close to the maximum growth rate in the absence of dust [8]. We observe in Fig. 1 that in the presence of dust a broad spectrum of $k_y$ values is excited.

Figure 2 contains the comparison of growth rates in dusty plasmas for different $P$ values (related to the dust density) and keeping the electron adiabaticity parameter $c_e$ constant. We can observe that the increase of $P$ has a stabilizing effect on the instability. One should remark that keeping $c_e$ constant and changing the parameter $P$ means that the values of the parallel wave numbers change when $P$ varies.

Let us now give some analytical results of the limiting solutions of the linear dispersion equation. For large values of $c_e$, we get from the system of Eqs. (49) the following equation for potential:

$$\frac{\partial}{\partial t}(\nabla^2\Psi-\Psi)-\frac{1}{(1+P-PL_d^{-1})}\frac{\partial\Psi}{\partial y}=\alpha(\Psi-z\chi)\ . \tag{51}$$

In the limit of a large charging frequency we can neglect the time derivative term in the charging Eq. (49c) and get

$$\Psi-z\chi=\gamma\Psi$$
$$\gamma=\left[\frac{(P+z)(\tau+z)+z(1+P)}{P(\tau+z)+z(1+P)(1+\tau+z)}\right]\ . \tag{52}$$

In deriving this expression we used the relation between the charging frequency and the particle-dust effective collision frequency (Eq. 21). From this result it is obvious that the coefficient $\gamma$ which is in front of $\Psi$ in the rhs of Eq. (51) is positive which means that the dust introduces a damping. One can easily solve Eq. (51) together with Eq. (49c) exactly without supposing that there is no delay in the charging process and show again that the presence of dust adds an additional damping due to the friction of the plasma particles on dust. We then consider another limit when the electrons are not adiabatic but the charging frequency is high enough to neglect the term with the time derivative in Eq. (49c). Then the set of the two linear equations describing the drift instability will have the form

$$\frac{\partial n}{\partial t}+\alpha\gamma n+\frac{\partial\Psi}{\partial y}=c_e(\Psi-n)\ ,$$
$$\frac{\partial}{\partial t}\nabla^2\Psi-\frac{\omega_{*d}}{1+P}\frac{\partial\Psi}{\partial y}=\frac{c_e}{1+P}(\Psi-n)\ , \tag{53}$$

where

$$\omega_{*d}=\frac{PL_d^{-1}}{(1+P-PL_d^{-1})}\ . \tag{54}$$

There are two solutions of the system (53). One of them describes the decaying mode due to the charging process. The other has a real frequency which in the limit $c_e \ll 1$ gives the dust drift wave frequency

$$\mathrm{Re}\,\omega\cong\frac{\omega_{*d}k_y}{k^2(1+P)}\ . \tag{55}$$

This branch has an imaginary part which for $c_e \ll 1$ can be found by the perturbation approach and is equal to

$$\mathrm{Im}\,\omega\cong c_e\frac{\dfrac{k_y^2\omega_{*d}}{k^2(1+P)^2}-\dfrac{k_y^2\omega_{*d}^2}{k^4(1+P)^3}-\dfrac{\alpha^2\gamma^2}{(1+P)}}{\alpha^2\gamma^2+\dfrac{k_y^2\omega_{*d}^2}{k^4(1+P)^2}}\ . \tag{56}$$

The necessary conditions for the instability are

$$\omega_{*d}>\alpha^2\gamma^2(1+P)\ , \tag{57}$$

$$k^2>\frac{\omega_{*d}}{(1+P)}\ . \tag{58}$$

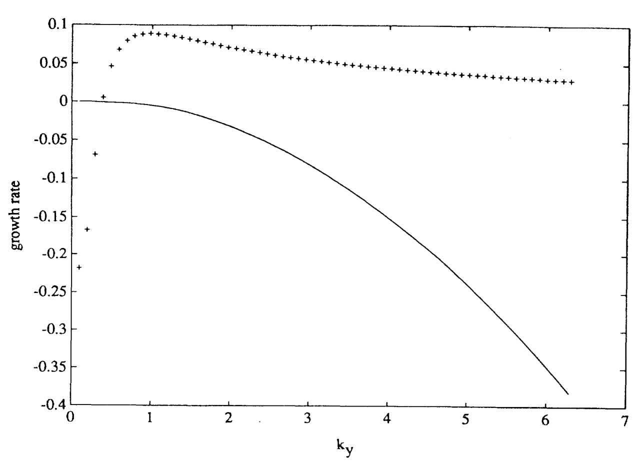


FIG. 1. Linear growth rates of dissipative drift waves vs $k_y$, in the presence of dust [dotted line, $P=0.1$, $c_e=0.1$, $L_d^{-1}=1$, $\nu_0=(\nu_{\mathrm{ch}}L_n)/(\omega_{ci}\rho_s)=1$] and for the correspondent adiabaticity parameter value in the absence of dust (solid line, $c_e=30$, $D=\nu=0.01$).

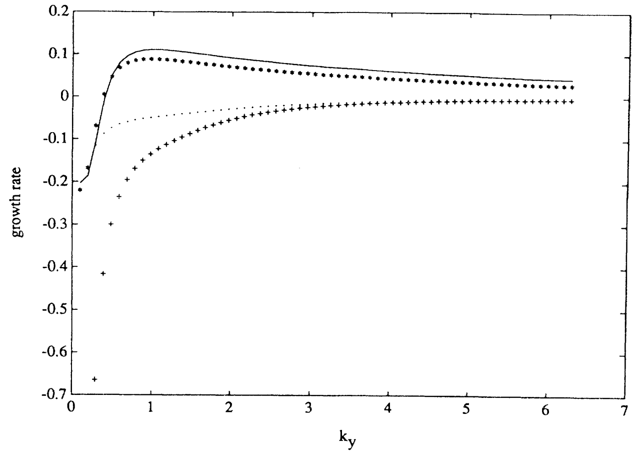


FIG. 2. Linear growth rates of the dissipative drift wave instability in dusty plasma vs $k_y$, for $c_e=0.1$, $L_d^{-1}=1$, $\nu_0=(\nu_{\mathrm{ch}}L_n)/(\omega_{ci}\rho_s)=1$, and the following $P$ values: $P=0.01$ (solid line), $P=0.1$ (stars), $P=0.5$ (dotted line), and $P=1$ (crosses).

## VI. SUMMARY

We have generalized the dissipative drift wave instabilities to the dusty plasmas taking into account only the collisions of the plasma particles with dust. This allows us to find a type of hydrodynamical description of the drift waves in which the dissipative processes are dominated by dust. We derived the full nonlinear set of equations for dissipative drift instability in dusty plasmas which constitutes a generalization of the well-known Hasegawa-Wakatani equations. The main qualitative effect introduced by the presence of dust is a high rate of dissipation due to the charging of dust particles. This dissipation produces a high nonadiabaticity in the longitudinal electron motion which increases the range of unstable drift waves. The dust produces a damping in the perpendicular direction which can have a stabilizing effect. Nevertheless the maximum growth rate of the instability in the presence of dust is close to the one obtained in the absence of dust but the range of the parameters for which it occurs is changed drastically.

The nonlinear behavior of drift waves described by Eqs. (49) is of interest for understanding the self-organization of the plasma turbulence in the presence of dust such as occurs in ionospheric plasma, astrophysical plasmas, and even in magnetically confined plasmas. In a future work we shall undertake numerical simulations of these equations for dusty plasmas.

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